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On neutron number dependence of $B(E1; 0_1^+ \to 1_1^-)$ reduced transition probability

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Abstract. A neutron number dependence of the $E1$ $0_1^+ \to 1_1^-$ reduced transition probability in spherical even-even nuclei is analysed within the Q-phonon approach in the fermionic space to describe the structure of collective states. Microscopic calculations of the $E1$ $0_1^+ \to 1_1^-$ transition matrix elements are carried out for the Xe isotopes based on the RPA for the ground-state wave function. A satisfactory description of the experimental data is obtained.

PACS. 21.60.Ev Collective models – 23.20.Js Multipole matrix elements

1 Introduction

The experimental data [1–7] demonstrate that the lowest-lying 1_1^- states in spherical nuclei have mainly the structure of two-phonon states arising as a result of the coupling of the collective quadrupole $|2_1^+\rangle$ and octupole $|3_1^-\rangle$ states: $|2_1^+ \otimes 3_1^-; 1^- M\rangle$. Thus, these states are isoscalar ones. However, important information about their structure comes from the $E1$ transitions which are characterized by the quantity $B(E1; 0_1^+ \to 1_1^-)$. The operator of the $E1$ transition is mainly an isovector operator and this is a very important circumstance, as this means that analysing $E1$ transition matrix elements we can obtain information about the proton-neutron structure of 1_1^- excitations.

Among the important experimental facts characterizing strong $E1$ transitions between the low-lying states is the following one. There is a minimum in the neutron number dependence of the matrix element $|\langle 0_1^+ \parallel \mathcal{M} \parallel 1_1^-\rangle|$ in the Nd, Sm and Ba isotopes when the number of neutrons N is equal to 78 or 86 [8–10]. Such a behavior of $B(E1; 1_1^- \to 0_1^+)$ as a function of the neutron number was discovered earlier in the RPA-based calculations [11]. A very schematic IBA-based analysis of the $E1$ transition $0_1^+ \to 1_1^-$ [12–14] showed that the appearance of the minimum in the neutron number dependence of $B(E1; 0_1^+ \to 1_1^-)$ is a result of cancelation of the proton and neutron contributions to an $E1$ transition matrix element for a number of valence neutrons. However, this phenomenological analysis cannot determine the number of neutrons at which $|\langle 0_1^+ \parallel \mathcal{M} \parallel 1_1^-\rangle|$ has a minimum. As it has been mentioned above, in Ba, Nd and Sm a minimum is located in the isotopes with four valence neutron particles or holes. However, the recently obtained results for the Xe isotopes [15] demonstrate the absence of a minimum in the neutron number dependence of $|\langle 0_1^+ \parallel \mathcal{M} \parallel 1_1^-\rangle|$ when the number of the valence neutron holes is equal to four. However, it is not improbable that this minimum exists at larger numbers of the neutron holes where there are no data.

This is the aim of the present paper to analyse the problem of appearance of a minimum in the neutron number dependence of $|\langle 0_1^+ \parallel \mathcal{M} \parallel 1_1^-\rangle|$ within the microscopic approach to description of the properties of the 1_1^- state.

In our preceding paper [14] we analysed the properties of the 1_1^- state in the framework of the fermionic Q-phonon description of the low-lying positive- and negative-parity collective states. The calculations performed showed that the fermionic Q-phonon approach was a good basis for the analysis of the properties of the low-lying collective states of both parities. The consideration below is based on this approach.

2 Neutron number dependence of $B(E1; 0_1^+ \to 1_1^-)$

In the Q-phonon approach formulated for the fermionic configurational space [14] the 1_1^- state is presented by the following expression:

$$|1_1^-, M\rangle = \mathcal{N}_{1_1^-} \left(\hat{Q}_2 \hat{Q}_3\right)_{1M} |0_1^+\rangle, \tag{1}$$

where $|0_1^+\rangle$ is the ground-state vector. The expression for the normalization coefficient $\mathcal{N}_{1_1^-}$ is given in [14], $\hat{Q}_{2\mu}$ and

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$\hat{Q}_{3\mu}$ are standard shell model quadrupole and octupole moment operators.

Since mainly the spherical nuclei have been considered, it is assumed that the ground states can be described in the RPA. An approximation of the ground-states wave vector by the RPA expression should be discussed in more detail. Let us do it by the example of Xe isotopes. Heavier Xe isotopes are spherical in their ground states. Therefore, for them an approximation of the ground state by the RPA expression is well justified. The lightest isotopes ^{124,126}Xe are treated in IBM as belonging to the $O(6)$ dynamical symmetry limit. Let us compare qualitatively the structure of the ground-state wave vectors in the $O(6)$ dynamical symmetry limit of IBM and in the RPA of the microscopical nuclear model. In the $O(6)$ limit of IBM

$$\begin{aligned}|0_1^+\rangle &= \sqrt{1-c_1^2-c_2^2-\ldots}\frac{1}{\sqrt{N!}}(s^+)^N|0\rangle \\ &+c_1\frac{1}{\sqrt{2}}(d^+d^+)_0\frac{1}{\sqrt{(N-2)!}}(s^+)^{N-2}|0\rangle \\ &+c_2(d^+d^+d^+d^+)_0\frac{1}{\sqrt{(N-4)!}}(s^+)^{N-4}|0\rangle+\ldots, \quad (2)\end{aligned}$$

where $|0\rangle$ is the boson vacuum and N is the maximum number of bosons. In the RPA with accuracy sufficient for our discussion

$$\begin{aligned}|0_1^+\rangle &= \sqrt{1-c_1^2-c_2^2-\ldots}|0\rangle + c_1\frac{1}{\sqrt{2}}(A_2^+A_2^+)_0|0\rangle \\ &+c_2(A_2^+A_2^+A_2^+A_2^+)_0|0\rangle+\ldots, \quad (3)\end{aligned}$$

where $|0\rangle$ is the quasiparticle vacuum, A_2^+ is the operator creating a collective superposition of two-quasiparticle states coupled to the angular momentum $L = 2$. We can say that there is a correspondence between the bifermion operator A_2^+ of the RPA and the boson operator d^+s of the IBM. Therefore, the ground-state wave functions in both approaches have a similar structure. A difference can arise from the value of the coefficient c_1 in (2) and (3). In the RPA the main component of the ground-state wave function is the first term in (3). However, in the IBM the second term in (2) can give a larger contribution. Using the consistent-Q IBM Hamiltonian we have found that $^{128-136}$Xe can be described using the RPA ground-state wave vector. However, in the case of ^{124,126}Xe the RPA underestimates the ground-state correlations and a ground state can be described approximately as a mixture of two lowest 0^+ RPA states. As a consequence, the strength of the $E1$ transition from the ground state will be fragmented between two 1^- states.

For the reduced matrix element of the $E1$ transition operator the following expression was derived in [14]:

$$\langle 1_1^- \parallel \mathcal{M}(E1) \parallel 0_1^+\rangle = (B_p - B_n)\ e\cdot \text{fm}, \quad (4)$$

where B_p and B_n represent the proton and neutron contributions to the $M1$ transition matrix element, respectively. The expressions for B_p and B_n are given in [14].

As it is seen from (4), the reduced matrix element $\langle 1_1^- \parallel \mathcal{M} \parallel 0_1^+\rangle$ is equal to the difference of the proton B_p and neutron B_n contributions. Let us consider separately a neutron number dependence of B_p and B_n. The results of calculations for the Xe isotopes are shown in fig. 1. It is seen that for the closed neutron shell ($N = 82$) the neutron contribution to the $E1$ transition matrix element is sufficiently smaller than the proton contribution. With increasing number of the neutron holes the neutron contribution to the $E1$ transition matrix element increases much faster than the proton contribution. However, the proton contribution increases also due to an increase in the collectivity of the low-lying states. As a result, the difference $(B_p - B_n)$ decreases with the neutron number and $|B_p - B_n|$ takes a minimum value at $A = 128$. With further decrease of A the $E1$ transition matrix element changes the sign and the modulus of the difference $|B_p - B_n|$ increases again. Thus, $|\langle 1_1^- \parallel \mathcal{M} \parallel 0_1^+\rangle|$ has a minimum at $A = 128$. A similar picture can be observed in the Nd and Sm isotopes if we start from the nucleus with the closed neutron shell $N = 82$ and then increase the number of the neutron holes. Thus, in the semimagic nucleus with the closed neutron shell the proton contribution to the $E1$ transition matrix element is the largest one. With increasing the number of the valence neutrons the neutron contribution increases. The proton and neutron contributions have opposite signs and for a number of neutrons the absolute value of the $E1$ transition matrix element takes a minimum. In the Nd and Sm isotopes this minimum takes place when the number of the valence neutrons or neutron holes is equal to four. According to our calculations in the Xe isotopes, this happens when the number of the neutron holes is equal to eight. It is clear that this number

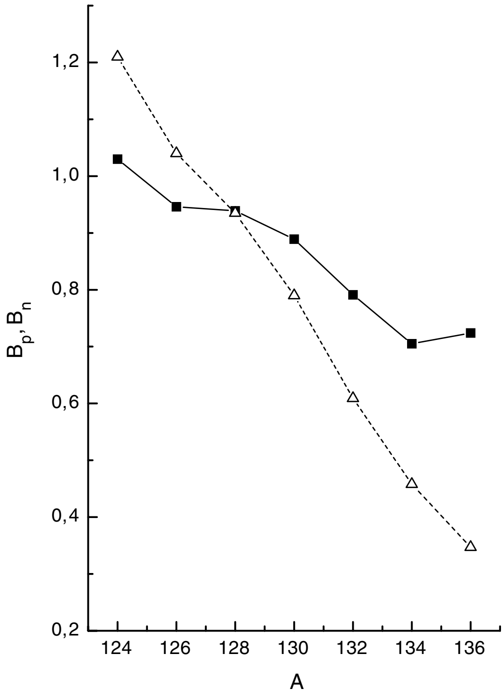

Fig. 1. Proton (solid line with full squares) and neutron (dashed line with open triangles) contributions to the $E1$ transition matrix element for the Xe isotopes (in units of 0.3 $e\cdot$fm).

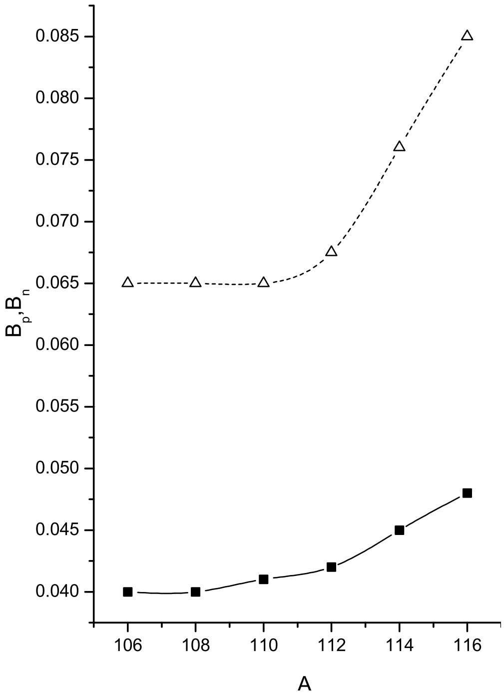

Fig. 2. The same as in fig. 1 but for the Cd isotopes.

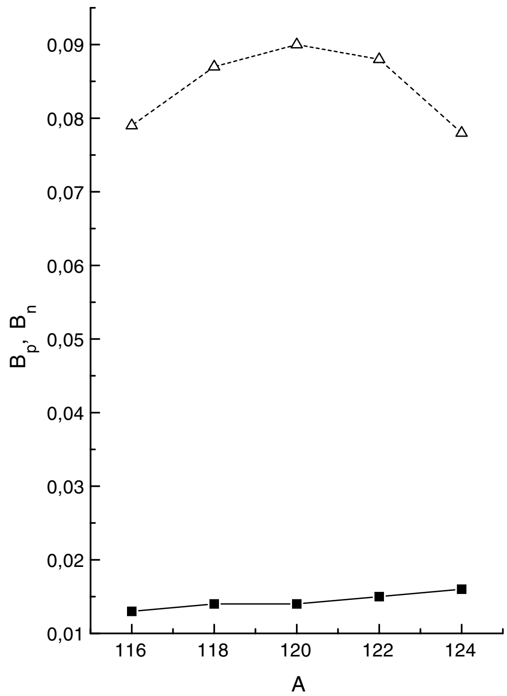

Fig. 3. The same as in fig. 1 but for the Sn isotopes.

can vary from element to element. With further increase in the number of the neutron holes the absolute value of the $E1$ transition matrix element in Xe isotopes increases again. Note, however, that when the neutron valence shell has a sufficiently large number of the valence particles, *i.e.*, when the neutron subshell is approximately half filled, as in $^{108-116}Cd_{60-68}$ and $^{116-124}Sn_{66-74}$ isotopes, the picture

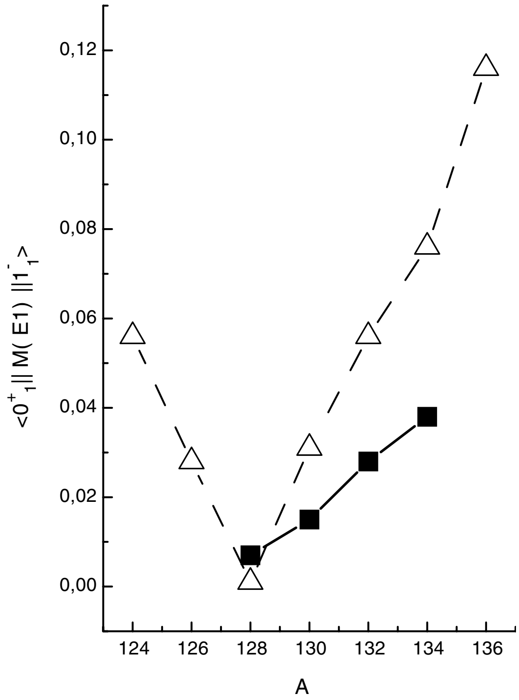

Fig. 4. The experimental (solid line with full squares) and calculated (dashed line with open triangles) electric dipole transition matrix elements for the Xe isotopes.

Table 1. The experimental (exp) and calculated (calc) electric dipole transition matrix elements for the Xe isotopes (in units of $e \cdot \mathrm{fm}$).

Nucleus	$\lvert\langle 1_1^- \parallel \mathcal{M}(E1) \parallel 0_1^+\rangle\rvert_{exp}$	$\lvert\langle 1_1^- \parallel \mathcal{M}(E1) \parallel 0_1^+\rangle\rvert_{calc}$
^{124}Xe	–	0.056
^{126}Xe	–	0.028
^{128}Xe	0.007	0.001
^{130}Xe	0.015	0.031
^{132}Xe	0.028	0.056
^{134}Xe	0.038	0.076
^{136}Xe	–	0.116

is changed. The proton and neutron contributions vary more or less in parallel. This is illustrated in figs. 2 and 3.

The results of calculations of $\lvert\langle 0_1^+ \parallel \mathcal{M} \parallel 1_1^-\rangle\rvert$ reduced matrix elements for the Xe isotopes are shown in fig. 4 and in table 1 together with the experimental data from [15]. The calculated $E1$ transition matrix elements decrease from ^{136}Xe to ^{128}Xe in agreement with the experimental data. However, in lighter Xe isotopes the experimental situation is unclear. Strong dipole transitions have been observed in these nuclei, but the parity of the excited dipole states was not determined. It is not improbable that they are due to the low-energy octupole strength expected for these isotopes [16]. If we assume that these states are 1^-, then we obtain from the experimental values of the reduced ground-state transition width Γ_0^{red} the following values of $\lvert\langle 0_1^+ \parallel \mathcal{M} \parallel 1_1^-\rangle\rvert$: 0.051 $e \cdot \mathrm{fm}$ for ^{124}Xe and 0.041 $e \cdot \mathrm{fm}$ for ^{126}Xe. The first value is close to the calculated one, the second one is somewhat higher.

As is seen from fig. 4 and table 1, the calculated $E1$ transition matrix elements in heavier Xe isotopes are two times larger than the experimental ones. This can be explained in the following way. The strength of the $E1$ transition $0_1^+ \to 1_1^-$ correlates with a magnitude of the ground-state correlations. The stronger the ground-state correlations, the larger the $E1$ transition matrix element $|\langle 0_1^+ \parallel \mathcal{M}(E1) \parallel 1_1^- \rangle|$. The ground-state correlations increase with decreasing energy of the low-lying collective states. For example, in the Xe isotopes considered in this paper a number of the neutron holes in the valence shell increases with decrease in the mass number A. Correspondingly, the energies of the 2_1^+ and 3_1^- states decrease and the ground-state correlations produced by the quadrupole and octupole forces increase with decreasing A. As a result, both the proton B_p and neutron B_n contributions to $|\langle 0_1^+ \parallel \mathcal{M}(E1) \parallel 1_1^- \rangle|$ increase with decreasing A. It seems that in a semimagic ^{136}Xe the ground-state correlations in the neutron subsystem are underestimated by the residual forces used. These residual forces overestimate an admixture to the ground-state wave function of the quasiparticle configurations with smaller energies and underestimate an admixture of the quasiparticle configurations with larger energies. Only latter neutron quasiparticle configurations are presented in ^{136}Xe because the neutron shell is closed. As a result, the value of B_n in ^{136}Xe is underestimated and the difference $(B_p - B_n)$ becomes too large.

In the Cd isotopes shown in fig. 2 both the quadrupole and octupole ground-state correlations increase with A. As a result, B_p and B_n increase with A. In the Sn isotopes shown in fig. 3 the proton contribution B_p is very small because a number of protons is semimagic. The neutron contribution B_n is almost independent of A. This happens because although the ground-state correlations produced by quadrupole forces increase with A the correlations produced by octupole forces decrease with A.

3 Conclusion

In this work a neutron number dependence of the $0_1^+ \to 1_1^-$ reduced transition probability in spherical even-even nuclei is analysed on the basis of the microscopic approach. Our calculations for the Xe isotopes have demonstrated that for the closed neutron shell ($N = 82$) the neutron contribution to the $E1$ transition matrix element is relatively small in comparison with the proton contribution. With increasing number of the neutron holes the neutron contribution to the $E1$ transition matrix element increases much faster than the proton contribution. As a result, a total transition matrix element which is a difference between the proton and the neutron contributions decreases with the neutron number and takes a minimum value at ^{128}Xe. With further decrease in A the $E1$ transition matrix element changes the sign and the modulus of $\langle 1_1^- \parallel E1 \parallel 0_1^+ \rangle$ increases again. Thus, it follows from our calculations that $B(E1; 0_1^+ \to 1_1^-)$ has a minimum at $A = 128$ for the Xe isotopes. The number of the valence neutrons at which $B(E1; 0_1^+ \to 1_1^-)$ has a minimum can vary from element to element. However, when the neutron subshell is approximately half filled the picture is changed, *i.e.* the proton and neutron contributions to the $E1$ transitional matrix element vary approximately in parallel without crossing.

The calculated $E1$ transition matrix elements decrease from ^{134}Xe to ^{128}Xe in agreement with the experimental data [2]. However, in lighter Xe isotopes the experimental situation is unclear.

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